Warning! If you are a physicist, you write everything differently.

Polar/Cylindrical: r becomes ρ, θ becomes φ.

Spherical: ρ becomes r, θ becomes φ. φ becomes θ

I assume there are good reasons for this

Example 1. (physical chemistry)

Let $f(\rho, \theta, \varphi) = \dfrac{1}{\sqrt{\pi a_0^3}} e^{-\rho/a_0}$

be the atomic orbital of a 1s hydrogen atom.

Here, "1" = energy level

"s" = shape of orbit (sphere)

"a_0" = Bohr radius

$\approx 5.2917 \times 10^{-11}$ m

The integral of the square of f over all of 3-space represents the probability of finding the hydrogen atom at all possible radii values ρ. Therefore, we should find

$$\int_0^{\infty}\int_0^{2\pi}\int_0^{\pi}\left(f(\rho,\theta,\varphi)\right)^2\rho^2\sin\varphi\,d\varphi\,d\theta\,d\rho$$

$$=1$$

Check:

$$\int_0^{\infty}\int_0^{2\pi}\int_0^{\pi}\left(\frac{e^{-\rho/a_0}}{\sqrt{\pi a_0^3}}\right)^2 \rho^2 \sin\varphi \, d\varphi \, d\theta \, d\rho$$

$$= 2\pi \int_0^{\infty} \frac{e^{-2\rho/a_0}\rho^2}{\pi a_0^3} d\rho \cdot \int_0^{\pi} \sin\varphi \, d\varphi$$

$$= \frac{2}{a_0^3}\int_0^{\infty} e^{-2\rho/a_0}\rho^2 d\rho \left(-\cos\varphi \Big|_0^{\pi}\right)$$

$$= \frac{4}{a_0^3}\int_0^{\infty} e^{-2\rho/a_0}\rho^2 d\rho$$

Then

$$\frac{4}{a_0^3}\int_0^\infty e^{-2\rho/a_0}\rho^2\,d\rho$$

$$=\frac{4}{a_0^3}\lim_{t\to\infty}\underbrace{\int_0^t e^{-2\rho/a_0}\rho^2\,d\rho}_{\text{integrate by parts}\atop \text{- tabular}}$$

u	dv
ρ^2	$e^{-2\rho/a_0}$
2ρ	$e^{-2\rho/a_0}\left(-a_0/2\right)$
2	$e^{-2\rho/a_0}\left(a_0/2\right)^2$
0	$e^{-2\rho/a_0}\left(-a_0/2\right)^3$

We get

$$\frac{4}{a_0^3} \lim_{t \to \infty} \left(-\frac{a_0}{2} e^{-2\rho/a_0}\right)\left(\rho^2 - \frac{2\rho a_0}{2} + \frac{a_0^2}{2}\right)\Bigg|_0^t$$

$$= \frac{4}{a_0^3}\left(\lim_{t \to \infty}\left(-\frac{a_0}{2} e^{-2t/a_0}\right)\left(t^2 - \frac{2 t a_0}{2} + \frac{a_0^2}{2}\right) - \left(-\frac{a_0^3}{4}\right)\right)$$

The limit is zero (l'Hopital's rule),

so we get

$$\frac{4}{a_0^3} \cdot \frac{a_0^3}{4} = \boxed{1}$$

Notes: 1) If the orbital is only dependent on ρ, you always end up with

$$4\pi \int_0^\infty \rho^2 f(\rho)^2 \, d\rho$$

2) Hydrogen is easiest (1 electron)

If you can ionize down to one electron, then the integrals that occur look much like hydrogen